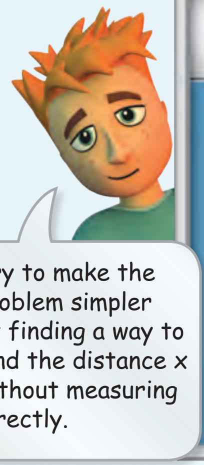
5-7 Inequalities in Two Triangles

Common Core State Standards / **Mathematics Florida Standards**

Extends G-CO.10 / **Extends MAFS.912.G-CO.3.10** Prove theorems about triangles. . . .

MP 1, MP 3

Objective To apply inequalities in two triangles



SOLVE IT! **Getting Ready!**

Think of a clock or watch that has an hour hand and a minute hand. As minutes pass, the distance between the tip of the hour hand and the tip of the minute hand changes. This distance is x in the figure at the right. What is the order of the times below from least to greatest length of x? How do you know?

1:00, 3:00, 5:00, 8:30, 1:30, 12:20

MATHEMATICAL PRACTICES In the Solve It, the hands of the clock and the segment labeled *x* form a triangle. As the time changes, the shape of the triangle changes, but the lengths of two of its sides do not change.

Essential Understanding In triangles that have two pairs of congruent sides, there is a relationship between the included angles and the third pair of sides.

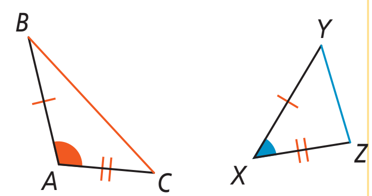
When you close a door, the angle between the door and the frame (at the hinge) gets smaller. The relationship between the measure of the hinge angle and the length of the opposite side is the basis for the SAS Inequality Theorem, also known as the Hinge Theorem.

take note

Theorem 5-13 The Hinge Theorem (SAS Inequality Theorem)

Theorem
If two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

If . . .
$m\angle A > m\angle X$



Then . . .
$BC > YZ$

You will prove Theorem 5-13 in Exercise 25.

Problem 1 Using the Hinge Theorem

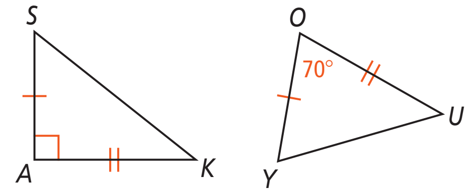
Plan

How do you apply the Hinge Theorem? After you identify the angles included between the pairs of congruent sides, locate the sides opposite those angles.

Multiple Choice **Which of the following statements must be true?**

Ⓐ $AS < YU$ Ⓒ $SK < YU$

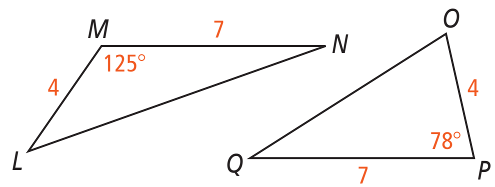
Ⓑ $SK > YU$ Ⓓ $AK = YU$



$\overline{SA} \cong \overline{YO}$ and $\overline{AK} \cong \overline{OU}$, so the triangles have two pairs of congruent sides. The included angles, $\angle A$ and $\angle O$, are not congruent. Since $m\angle A > m\angle O$, $SK > YU$ by the Hinge Theorem. The correct answer is B.

Got It? **1. a.** What inequality relates LN and OQ in the figure at the right?

b. Reasoning In $\triangle ABC$, $AB = 3$, $BC = 4$, and $CA = 6$. In $\triangle PQR$, $PQ = 3$, $QR = 5$, and $RP = 6$. How can you use indirect reasoning to explain why $m\angle P > m\angle A$?



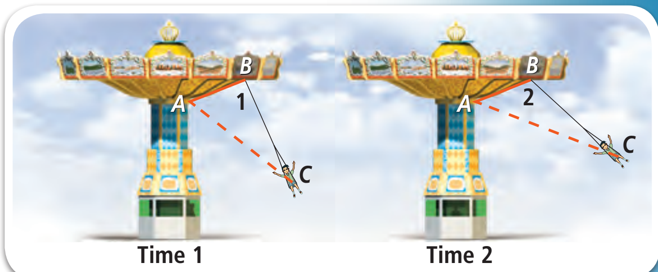
Problem 2 Applying the Hinge Theorem

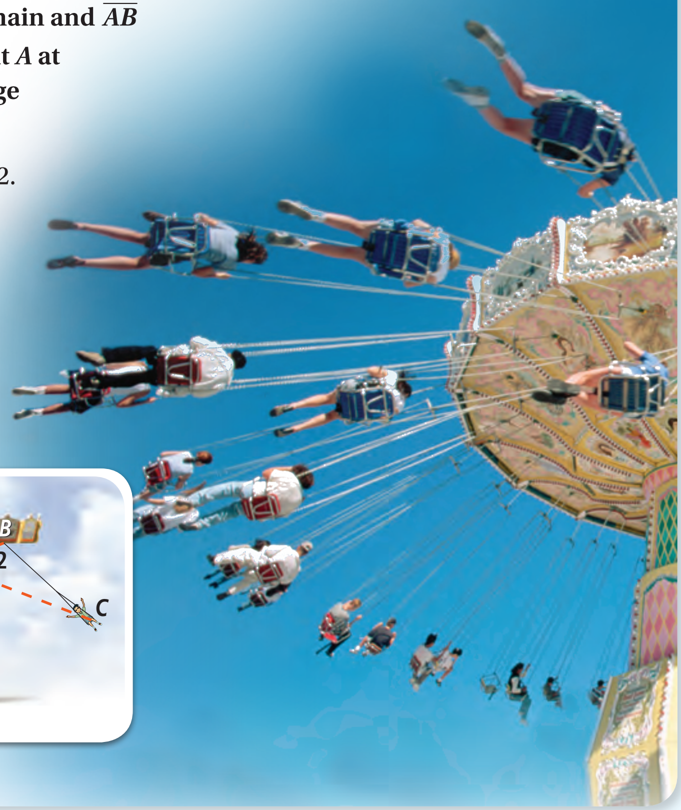
Think

For △ABC, which side lengths are the same at Time 1 and Time 2? The lengths of the chain and $\overline{AB}$ do not change. So, AB and BC are the same at Time 1 and Time 2.

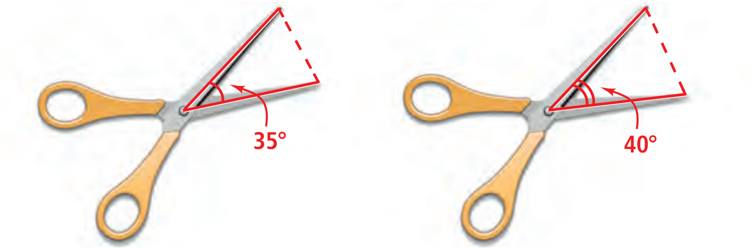
Swing Ride **The diagram below shows the position of a swing at two different times. As the speed of the swing ride increases, the angle between the chain and $\overline{AB}$ increases. Is the rider farther from point A at Time 1 or Time 2? Explain how the Hinge Theorem justifies your answer.**

The rider is farther from point A at Time 2. The lengths of $\overline{AB}$ and $\overline{BC}$ stay the same throughout the ride. Since the angle formed at Time 2 ($\angle 2$) is greater than the angle formed at Time 1 ($\angle 1$), you can use the Hinge Theorem to conclude that $\overline{AC}$ at Time 2 is longer than $\overline{AC}$ at Time 1.



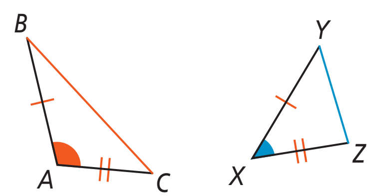


Got It? **2.** The diagram below shows a pair of scissors in two different positions. In which position is the distance between the tips of the two blades greater? Use the Hinge Theorem to justify your answer.



The Converse of the Hinge Theorem is also true. The proof of the converse is an indirect proof.

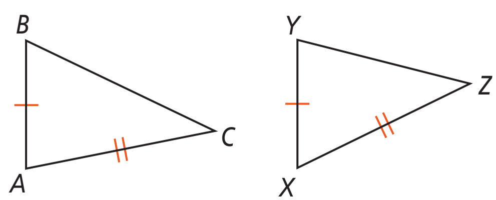
take note — Theorem 5-14 Converse of the Hinge Theorem (SSS Inequality)

Theorem	If . . .	Then . . .
If two sides of one triangle are congruent to two sides of another triangle, and the third sides are not congruent, then the larger included angle is opposite the longer third side.	$BC > YZ$ 	$m\angle A > m\angle X$

Proof Indirect Proof of the Converse of the Hinge Theorem (SSS Inequality)

Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $BC > YZ$

Prove: $m\angle A > m\angle X$



Step 1 Assume temporarily that $m\angle A \ngtr m\angle X$. This means either $m\angle A < m\angle X$ or $m\angle A = m\angle X$.

Step 2 If $m\angle A < m\angle X$, then $BC < YZ$ by the Hinge Theorem. This contradicts the given information that $BC > YZ$. Therefore, the assumption that $m\angle A < m\angle X$ must be false.

If $m\angle A = m\angle X$, then $\triangle ABC \cong \triangle XYZ$ by SAS. If the two triangles are congruent, then $BC = YZ$ because corresponding parts of congruent triangles are congruent. This contradicts the given information that $BC > YZ$. Therefore, the assumption that $m\angle A = m\angle X$ must be false.

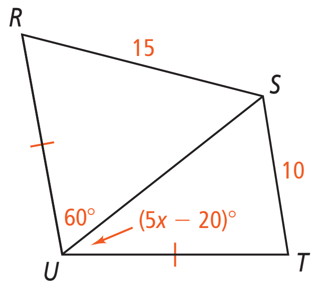
Step 3 The temporary assumption that $m\angle A \ngtr m\angle X$ is false. Therefore, $m\angle A > m\angle X$.

Problem 3 Using the Converse of the Hinge Theorem

Plan

How do you put upper and lower limits on the value of x?
Use the largest possible value of $m\angle TUS$ as the upper limit for $5x - 20$ and the smallest possible value of $m\angle TUS$ as the lower limit for $5x - 20$.

Algebra **What is the range of possible values for x?**



Step 1 Find an upper limit for the value of x. $\overline{UT} \cong \overline{UR}$ and $\overline{US} \cong \overline{US}$, so $\triangle TUS$ and $\triangle RUS$ have two pairs of congruent sides. $RS > TS$, so you can use the Converse of the Hinge Theorem to write an inequality.

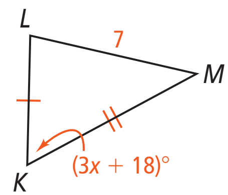
$m\angle RUS > m\angle TUS$	Converse of the Hinge Theorem
$60 > 5x - 20$	Substitute.
$80 > 5x$	Add 20 to each side.
$16 > x$	Divide each side by 5.

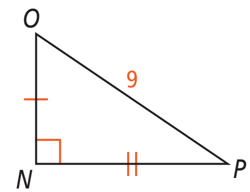
Step 2 Find a lower limit for the value of x.

$m\angle TUS > 0$	The measure of an angle of a triangle is greater than 0.
$5x - 20 > 0$	Substitute.
$5x > 20$	Add 20 to each side.
$x > 4$	Divide each side by 5.

Rewrite $16 > x$ and $x > 4$ as $4 < x < 16$.

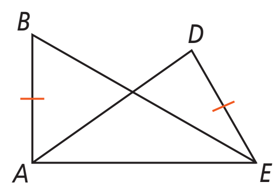
Got It? **3.** What is the range of possible values for x in the figure at the right?





Proof

Problem 4 Proving Relationships in Triangles



Given: $BA = DE$, $BE > DA$

Prove: $m\angle BAE > m\angle BEA$

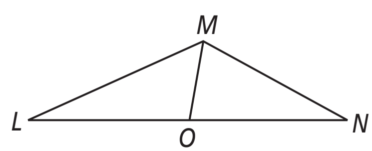
Statement	Reasons
1) $BA = DE$	**1)** Given
2) $AE = AE$	**2)** Reflexive Property of Equality
3) $BE > DA$	**3)** Given
4) $m\angle BAE > m\angle DEA$	**4)** Converse of the Hinge Theorem
5) $m\angle DEA = m\angle DEB + m\angle BEA$	**5)** Angle Addition Postulate
6) $m\angle DEA > m\angle BEA$	**6)** Comparison Property of Inequality
7) $m\angle BAE > m\angle BEA$	**7)** Transitive Property of Inequality

Think

How do you know $m\angle BAE > m\angle BEA$?
Use the Transitive Property of Inequality on the inequalities in Statements 4 and 6.

Got It? **4. Given:** $m\angle MON = 80$, O is the midpoint of $\overline{LN}$

Prove: $LM > MN$

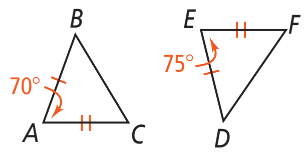


Lesson Check

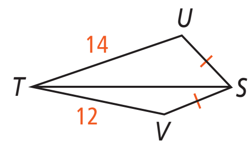
Do you know HOW?

Write an inequality relating the given side lengths or angle measures.

1. *FD* and *BC*



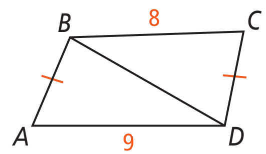
2. $m\angle UST$ and $m\angle VST$



Do you UNDERSTAND? MATHEMATICAL PRACTICES

3. Vocabulary Explain why *Hinge Theorem* is an appropriate name for Theorem 5-13.

4. Error Analysis From the figure at the right, your friend concludes that $m\angle BAD > m\angle BCD$. How would you correct your friend's mistake?



5. Compare and Contrast How are the Hinge Theorem and the SAS Congruence Postulate similar?

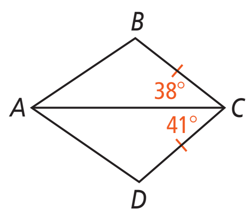
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

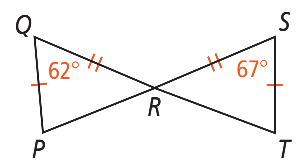
Write an inequality relating the given side lengths. If there is not enough information to reach a conclusion, write *no conclusion*.

See Problem 1.

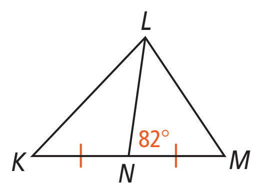
6. *AB* and *AD*



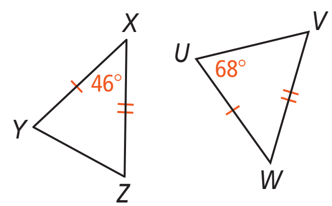
7. *PR* and *RT*



8. *LM* and *KL*

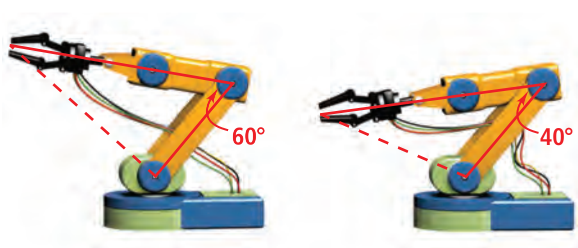


9. *YZ* and *UV*



10. The diagram below shows a robotic arm in two different positions. In which position is the tip of the robotic arm closer to the base? Use the Hinge Theorem to justify your answer.

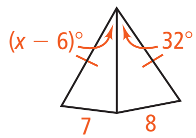
See Problem 2.



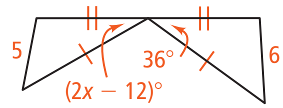
Algebra Find the range of possible values for each variable.

See Problem 3.

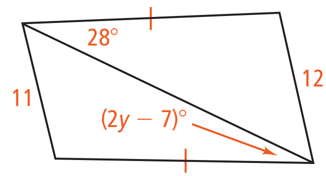
11.



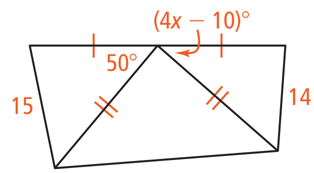
12.



13.



14.

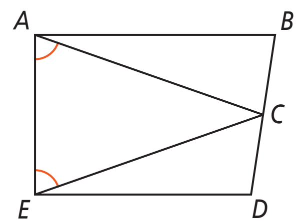


15. Developing Proof Complete the following proof.

See Problem 4.

Given: C is the midpoint of $\overline{BD}$,
$m\angle EAC = m\angle AEC$,
$m\angle BCA > m\angle DCE$

Prove: $AB > ED$



Statements	Reasons
1) $m\angle EAC = m\angle AEC$	1) Given
2) $AC = EC$	2) **a.** ?
3) C is the midpoint of $\overline{BD}$.	3) **b.** ?
4) $\overline{BC} \cong \overline{CD}$	4) **c.** ?
5) **d.** ?	5) $\cong$ segments have $=$ length.
6) $m\angle BCA > m\angle DCE$	6) **e.** ?
7) $AB > ED$	7) **f.** ?

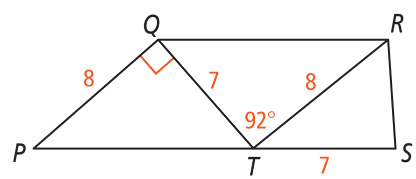
B Apply

Copy and complete with $>$ or $<$. Explain your reasoning.

16. PT ■ QR

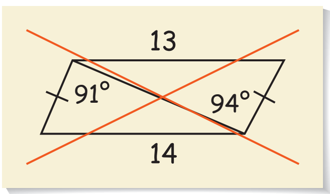
17. $m\angle QTR$ ■ $m\angle RTS$

18. PT ■ RS



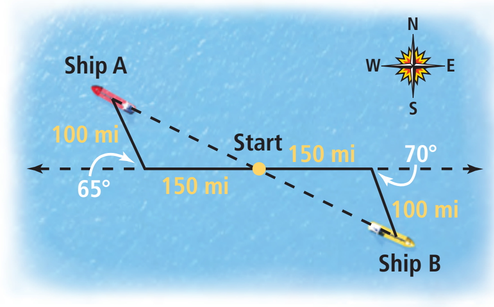
19. a. Error Analysis Your classmate draws the figure at the right. Explain why the figure cannot have the labeled dimensions.

b. Open-Ended Describe a way you could change the dimensions to make the figure possible.



20. Think About a Plan Ship A and Ship B leave from the same point in the ocean. Ship A travels 150 mi due west, turns 65° toward north, and then travels another 100 mi. Ship B travels 150 mi due east, turns 70° toward south, and then travels another 100 mi. Which ship is farther from the starting point? Explain.

- How can you use the given angle measures?
- How does the Hinge Theorem help you to solve this problem?



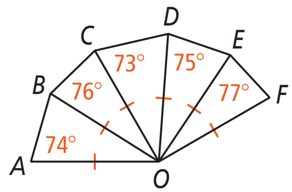
21. Which of the following lists the segment lengths in order from least to greatest?

Ⓐ *CD, AB, DE, BC, EF*

Ⓑ *EF, DE, AB, BC, CD*

Ⓒ *BC, DE, EF, AB, CD*

Ⓓ *EF, BC, DE, AB, CD*

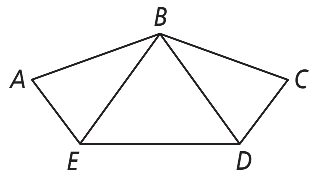


22. Reasoning The legs of a right isosceles triangle are congruent to the legs of an isosceles triangle with an 80° vertex angle. Which triangle has a greater perimeter? How do you know?

23. Use the figure at the right.
Proof

Given: $\triangle ABE$ is isosceles with vertex $\angle B$,
$\triangle ABE \cong \triangle CBD$,
$m\angle EBD > m\angle ABE$

Prove: $ED > AE$



Challenge

24. Coordinate Geometry $\triangle ABC$ has vertices $A(0, 7)$, $B(-1, -2)$, $C(2, -1)$, and $O(0, 0)$. Show that $m\angle AOB > m\angle AOC$.

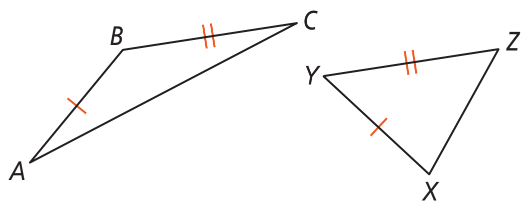
25. Use the plan below to complete a proof of the Hinge Theorem: If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the longer third side is opposite the larger included angle.
Proof

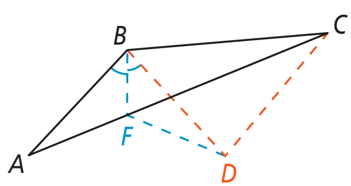
Given: $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $m\angle B > m\angle Y$

Prove: $AC > XZ$

Plan for proof:

- Copy $\triangle ABC$. Locate point D outside $\triangle ABC$ so that $m\angle CBD = m\angle ZYX$ and $BD = YX$. Show that $\triangle DBC \cong \triangle XYZ$.
- Locate point F on $\overline{AC}$, so that $\overline{BF}$ bisects $\angle ABD$.
- Show that $\triangle ABF \cong \triangle DBF$ and that $\overline{AF} \cong \overline{DF}$.
- Show that $AC = FC + DF$.
- Use the Triangle Inequality Theorem to write an inequality that relates DC to the lengths of the other sides of $\triangle FCD$.
- Relate DC and XZ.



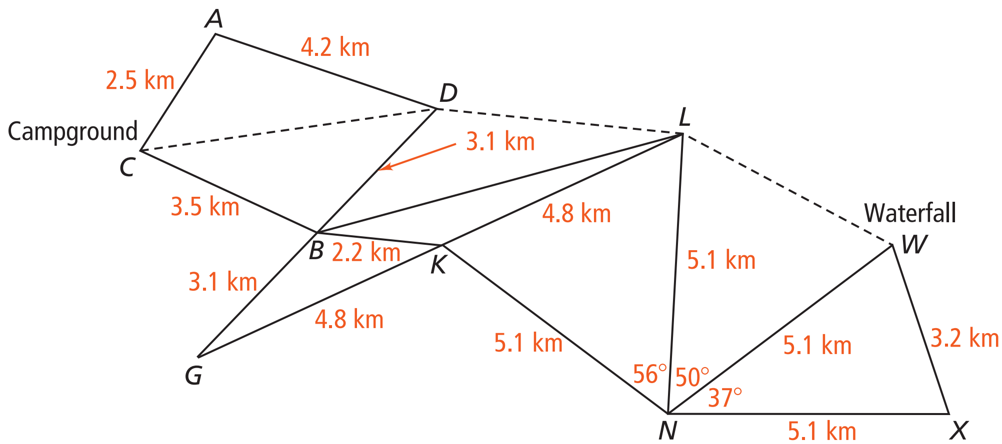


PERFORMANCE TASK

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 2

Look at the trail map from page 283, shown again below. In the Apply What You've Learned sections in Lessons 5-1 and 5-6, you worked with the first two segments of the hike from the campground to the waterfall. Now you will consider the last segment of the hike.



a. Write an inequality that shows an upper bound for the length of $\overline{LW}$. Explain your reasoning.

b. Write an inequality that shows a lower bound for the length of $\overline{LW}$. Explain your reasoning.

c. Use your answers from parts (a) and (b) to write a compound inequality that shows the range of possible lengths of $\overline{LW}$.

5 Pull It All Together

Completing the Performance Task

Look back at your results from the Apply What You've Learned sections in Lessons 5-1, 5-6, and 5-7. Use the work you did to complete the following.

1. Solve the problem in the Task Description on page 283 by finding a range, in kilometers, for the length of the group's hike from the campground to the waterfall. Show all your work and explain each step of your solution.

2. **Reflect** Choose one of the Mathematical Practices below and explain how you applied it in your work on the Performance Task.

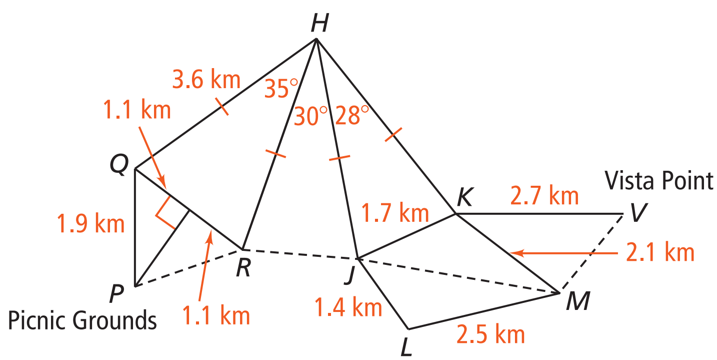
   MP 2: Reason abstractly and quantitatively.

   MP 3: Construct viable arguments and critique the reasoning of others.

   MP 7: Look for and make use of structure.

On Your Own

The hikers are planning their next hike, from the picnic grounds to the vista point in the map below. Once again, the hike is shown with a dashed line and some of the trail lengths are missing on the map.



Find a range, in kilometers, for the length of the group's hike from the picnic grounds to the vista point.

5 Chapter Review

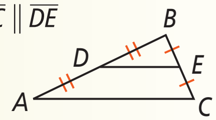
Connecting BIG ideas and Answering the Essential Questions

1 Coordinate Geometry
Use parallel and perpendicular lines, and the slope, midpoint, and distance formulas to find intersection points and unknown lengths.

2 Measurement
Use theorems about perpendicular bisectors, angle bisectors, medians, and altitudes to find points of concurrency, angle measures, and segment lengths.

3 Reasoning and Proof
You can write an indirect proof by showing that a temporary assumption is false.

Midsegments of Triangles (Lesson 5-1)
If $\overline{DE}$ is a midsegment, then $\overline{AC} \parallel \overline{DE}$ and $DE = \frac{1}{2}AC$.



Concurrent Lines and Segments in Triangles (Lessons 5-2, 5-3, and 5-4)

Concurrent Lines and Segments	Intersection
• perpendicular bisectors	• circumcenter
• angle bisectors	• incenter
• medians	• centroid
• lines containing altitudes	• orthocenter

Indirect Proof (Lesson 5-5)
1) Assume temporarily the opposite of what you want to prove.
2) Show that this temporary assumption leads to a contradiction.
3) Conclude that what you want to prove is true.

Inequalities in Triangles (Lessons 5-6 and 5-7)
Use indirect reasoning to prove that the longer of two sides of a triangle lies opposite the larger angle, and to prove the Converse of the Hinge Theorem.

Chapter Vocabulary

- altitude of a triangle (p. 310)
- centroid of a triangle (p. 309)
- circumcenter of a triangle (p. 301)
- circumscribed about (p. 301)
- concurrent (p. 301)
- distance from a point to a line (p. 294)
- equidistant (p. 292)
- incenter of a triangle (p. 303)
- indirect proof (p. 317)
- indirect reasoning (p. 317)
- inscribed in (p. 303)
- median of a triangle (p. 309)
- midsegment of a triangle (p. 285)
- orthocenter of a triangle (p. 311)
- point of concurrency (p. 301)

Choose the correct vocabulary term to complete each sentence.

1. A *(centroid, median)* of a triangle is a segment from a vertex of the triangle to the midpoint of the side opposite the vertex.

2. The length of the perpendicular segment from a point to a line is the *(midsegment, distance from a point to the line)*.

3. The *(circumcenter, incenter)* of a triangle is the point of concurrency of the angle bisectors of the triangle.

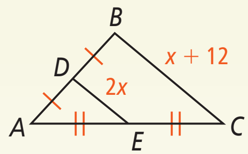
5-1 Midsegments of Triangles

Quick Review

A **midsegment of a triangle** is a segment that connects the midpoints of two sides. A midsegment is parallel to the third side and is half as long.

Example

Algebra **Find the value of x.**



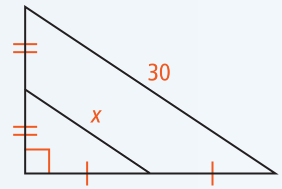
$\overline{DE}$ is a midsegment because D and E are midpoints.

$DE = \frac{1}{2}BC$	△ Midsegment Theorem
$2x = \frac{1}{2}(x + 12)$	Substitute.
$4x = x + 12$	Simplify.
$3x = 12$	Subtract x from each side.
$x = 4$	Divide each side by 3.

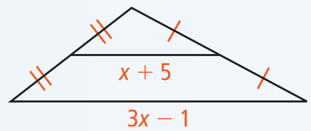
Exercises

Algebra **Find the value of x.**

4.



5.



6. $\triangle ABC$ has vertices $A(0, 0)$, $B(2, 2)$, and $C(5, -1)$. Find the coordinates of L, the midpoint of $\overline{AC}$, and M, the midpoint of $\overline{BC}$. Verify that $\overline{LM} \parallel \overline{AB}$ and $LM = \frac{1}{2}AB$.

5-2 Perpendicular and Angle Bisectors

Quick Review

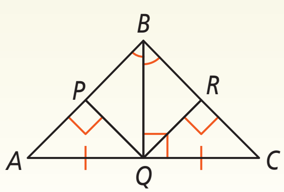
The **Perpendicular Bisector Theorem** together with its converse states that P is equidistant from A and B if and only if P is on the perpendicular bisector of $\overline{AB}$.

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line.

The **Angle Bisector Theorem** together with its converse states that P is equidistant from the sides of an angle if and only if P is on the angle bisector.

Example

In the figure, $QP = 4$ and $AB = 8$. Find QR and CB.



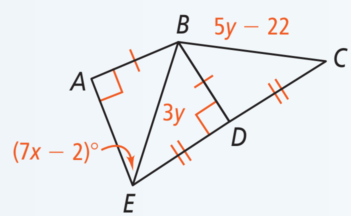
Q is on the bisector of $\angle ABC$, so $QR = QP = 4$.

B is on the perpendicular bisector of $\overline{AC}$, so $CB = AB = 8$.

Exercises

7. Writing Describe how to find all the points on a baseball field that are equidistant from second base and third base.

In the figure, $m\angle DBE = 50$. Find each of the following.



8. $m\angle BED$

9. $m\angle BEA$

10. x

11. y

12. BE

13. BC

5-3 Bisectors in Triangles

Quick Review

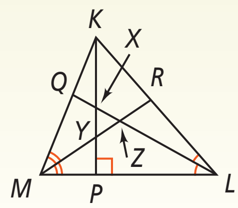
When three or more lines intersect in one point, they are **concurrent.**

- The point of concurrency of the perpendicular bisectors of a triangle is the **circumcenter of the triangle.**
- The point of concurrency of the angle bisectors of a triangle is the **incenter of the triangle.**

Example

Identify the incenter of the triangle.

The incenter of a triangle is the point of concurrency of the angle bisectors. $\overline{MR}$ and $\overline{LQ}$ are angle bisectors that intersect at Z. So, Z is the incenter.



Exercises

Find the coordinates of the circumcenter of $\triangle DEF$.

14. $D(6, 0)$, $E(0, 6)$, $F(-6, 0)$

15. $D(0, 0)$, $E(6, 0)$, $F(0, 4)$

16. $D(5, -1)$, $E(-1, 3)$, $F(3, -1)$

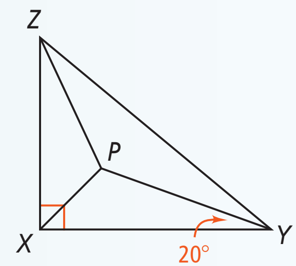
17. $D(2, 3)$, $E(8, 3)$, $F(8, -1)$

P is the incenter of $\triangle XYZ$. Find the indicated angle measure.

18. $m\angle PXY$

19. $m\angle XYZ$

20. $m\angle PZX$



5-4 Medians and Altitudes

Quick Review

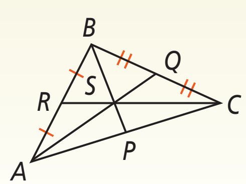
A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side. An **altitude of a triangle** is a perpendicular segment from a vertex to the line containing the opposite side.

- The point of concurrency of the medians of a triangle is the **centroid of the triangle.** The centroid is two thirds the distance from each vertex to the midpoint of the opposite side.
- The point of concurrency of the altitudes of a triangle is the **orthocenter of the triangle.**

Example

If $PB = 6$, what is SB?

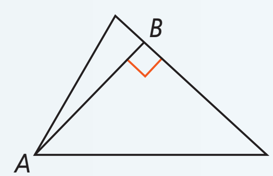
S is the centroid because $\overline{AQ}$ and $\overline{CR}$ are medians. So, $SB = \frac{2}{3}PB = \frac{2}{3}(6) = 4$.



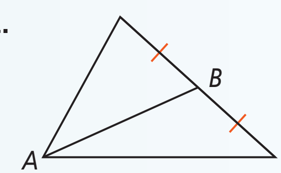
Exercises

Determine whether $\overline{AB}$ is a *median*, an *altitude*, or *neither*. Explain.

21.



22.



23. $\triangle PQR$ has medians $\overline{QM}$ and $\overline{PN}$ that intersect at Z. If $ZM = 4$, find QZ and QM.

$\triangle ABC$ has vertices $A(2, 3)$, $B(-4, -3)$, and $C(2, -3)$. Find the coordinates of each point of concurrency.

24. centroid

25. orthocenter

5-5 Indirect Proof

Quick Review

In an **indirect proof,** you first assume temporarily the opposite of what you want to prove. Then you show that this temporary assumption leads to a contradiction.

Example

Which two statements contradict each other?

I. The perimeter of $\triangle ABC$ is 14.

II. $\triangle ABC$ is isosceles.

III. The side lengths of $\triangle ABC$ are 3, 5, and 6.

An isosceles triangle can have a perimeter of 14.

The perimeter of a triangle with side lengths 3, 5, and 6 is 14.

An isosceles triangle must have two sides of equal length. Statements II and III contradict each other.

Exercises

Write a convincing argument that uses indirect reasoning.

26. The product of two numbers is even. Show that at least one of the numbers must be even.

27. Two lines in the same plane are not parallel. Show that a third line in the plane must intersect at least one of the two lines.

28. Show that a triangle can have at most one obtuse angle.

29. Show that an equilateral triangle cannot have an obtuse angle.

30. The sum of three integers is greater than 9. Show that one of the integers must be greater than 3.

5-6 and 5-7 Inequalities in Triangles

Quick Review

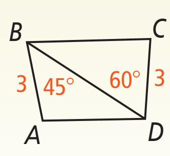
For any triangle,

- the measure of an exterior angle is greater than the measure of each of its remote interior angles
- if two sides are not congruent, then the larger angle lies opposite the longer side
- if two angles are not congruent, then the longer side lies opposite the larger angle
- the sum of any two side lengths is greater than the third

The **Hinge Theorem** states that if two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

Example

Which is greater, BC or AD?



$\overline{BA} \cong \overline{CD}$ and $\overline{BD} \cong \overline{DB}$, so $\triangle ABD$ and $\triangle CDB$ have two pairs of congruent corresponding sides. Since $60 > 45$, you know $BC > AD$ by the Hinge Theorem.

Exercises

31. In $\triangle RST$, $m\angle R = 70$ and $m\angle S = 80$. List the sides of $\triangle RST$ in order from shortest to longest.

Is it possible for a triangle to have sides with the given lengths? Explain.

32. 5 in., 8 in., 15 in.

33. 10 cm, 12 cm, 20 cm

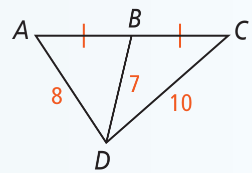
34. The lengths of two sides of a triangle are 12 ft and 13 ft. Find the range of possible lengths for the third side.

Use the figure below. Complete each statement with >, <, or =.

35. $m\angle BAD$ ■ $m\angle ABD$

36. $m\angle CBD$ ■ $m\angle BCD$

37. $m\angle ABD$ ■ $m\angle CBD$



5 Chapter Test

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Do you know HOW?

Find the coordinates of the circumcenter of $\triangle ABC$.

1. $A(3, -1), B(-2, -1), C(3, -8)$

2. $A(0, 5), B(-4, 5), C(-4, -3)$

Find the coordinates of the orthocenter of $\triangle ABC$.

3. $A(-1, -1), B(-1, 5), C(-4, -1)$

4. $A(0, 0), B(5, 0), C(5, 3)$

Identify the two statements that contradict each other.

5. **I.** $\triangle PQR$ is a right triangle.
II. $\triangle PQR$ is an obtuse triangle.
III. $\triangle PQR$ is scalene.

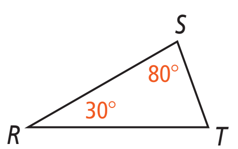
6. **I.** $\angle DOS \cong \angle CAT$
II. $\angle DOS$ and $\angle CAT$ are vertical.
III. $\angle DOS$ and $\angle CAT$ are adjacent.

7. If $AB = 9$, $BC = 4\frac{1}{2}$, and $AC = 12$, list the angles of $\triangle ABC$ from smallest to largest.

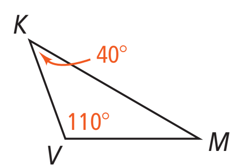
8. Point P is inside $\triangle ABC$ and equidistant from all three sides. If $m\angle ABC = 60$, what is $m\angle PBC$?

List the sides from shortest to longest.

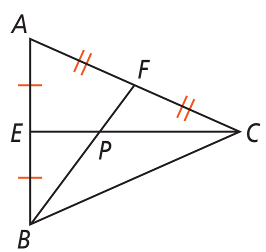
9.



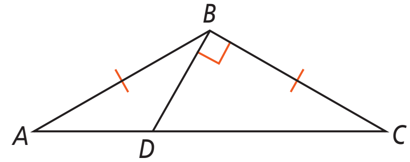
10.



11. In $\triangle ABC$, $EP = 4$. What is PC?

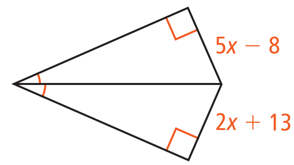


12. Which is greater, AD or DC? Explain.

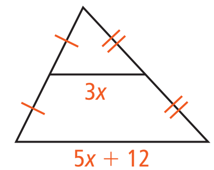


Algebra **Find the value of x.**

13.

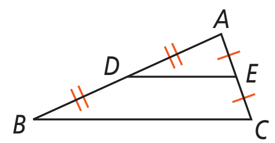


14.



Do you UNDERSTAND?

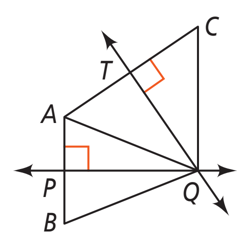
15. What can you conclude from the diagram at the right? Justify your answer.



16. **Reasoning** In $\triangle ABC$, $BC > BA$. Draw $\triangle ABC$ and the median $\overline{BD}$. Use the Converse of the Hinge Theorem to explain why $\angle BDC$ is obtuse.

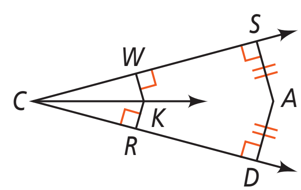
17. **Given:** $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$. $\overleftrightarrow{QT}$ is the perpendicular bisector of $\overline{AC}$.

Prove: $QC = QB$



18. **Writing** Use indirect reasoning to explain why the following statement is true: If an isosceles triangle is obtuse, then the obtuse angle is the vertex angle.

19. In the figure, $WK = KR$. What can you conclude about point A? Explain.



Common Core Cumulative Standards Review

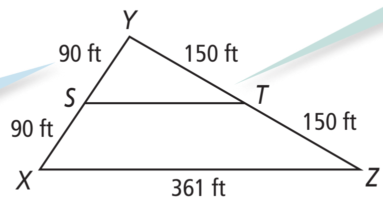
5

ASSESSMENT

TIPS FOR SUCCESS

Some test questions ask you to find missing measurements. Read the sample question at the right. Then follow the tips to answer it.

The diagram below shows the walkways in a triangular park. What is the distance from point S to point Z using the walkways $\overline{ST}$ and $\overline{TZ}$?



Ⓐ 180.5 ft
Ⓑ 330.5 ft
Ⓒ 390 ft
Ⓓ 451 ft

TIP 1

You need to find the length of $\overline{ST}$ and add it to 150 ft, the length of $\overline{TZ}$.

TIP 2

From the dimensions given in the diagram, you can conclude that S and T are midpoints of sides of △XYZ. So you can use the Triangle Midsegment Theorem to find ST.

Think It Through

$\overline{ST}$ is a midsegment of $\triangle XYZ$. By the Triangle Midsegment Theorem, you know $ST = \frac{1}{2}(XZ) = \frac{1}{2}(361)$, or 180.5 ft. The distance from point S to point Z using walkways $\overline{ST}$ and $\overline{TZ}$ is $ST + TZ = 180.5 + 150$, or 330.5 ft. The correct answer is B.

Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Choose the correct term to complete each sentence.

I. The (*inverse*, *converse*) of a conditional statement negates both the hypothesis and the conclusion.

II. The lines containing the altitudes of a triangle are concurrent at the (*orthocenter*, *centroid*) of the triangle.

III. The side opposite the vertex angle of an isosceles triangle is the (*hypotenuse*, *base*).

IV. The linear equation $y = mx + b$ is in (*slope-intercept form*, *point-slope form*).

Selected Response

Read each question. Then write the letter of the correct answer on your paper.

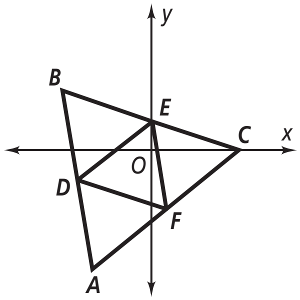
1. One side of a triangle has length 6 in. and another side has length 3 in. Which is the greatest possible value for the length of the third side?

Ⓐ 3 in.
Ⓑ 6 in.
Ⓒ 8 in.
Ⓓ 9 in.

2. $\triangle ABC$ is an equilateral triangle. Which is NOT a true statement about $\triangle ABC$?

Ⓕ All three sides have the same length.
Ⓖ $\triangle ABC$ is isosceles.
Ⓗ $\triangle ABC$ is equiangular.
Ⓘ The measure of $\angle A$ is 50.

3. In the figure below, $\triangle ABC$ has vertices $A(-2, -4)$, $B(-3, 2)$, and $C(3, 0)$. D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$, and F is the midpoint of $\overline{AC}$. What are the coordinates of the vertices of $\triangle DEF$?



(A) $D(-2.5, -1)$, $E(0, 1)$, $F(0.5, -2)$

(B) $D(-2, -1)$, $E(0, 0.5)$, $F(0.5, -1)$

(C) $D(-2, -1.5)$, $E(0, 1.5)$, $F(0.8, -2)$

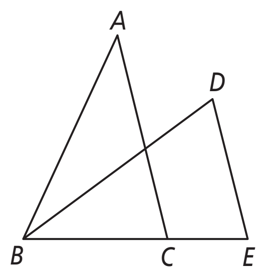
(D) $D(-2.2, -0.8)$, $E(0, 1.5)$, $F(0.5, -2)$

4. A square and a rectangle have the same area. The square has side length 8 in. The length of the rectangle is four times its width. What is the length of the rectangle?

(F) 4 in. (H) 32 in.

(G) 16 in. (I) 64 in.

5. In the figure below, $\angle A \cong \angle DBE$ and $\overline{AB} \cong \overline{BE}$. What additional information do you need in order to prove a pair of triangles congruent by AAS?



(A) $\angle ABC \cong \angle BED$

(B) C is the midpoint of $\overline{BE}$.

(C) $\angle ACB \cong \angle BDE$

(D) $\angle ABC \cong \angle BDE$

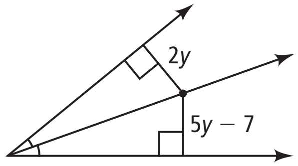
6. What is the value of y in the figure below?

(F) $\frac{3}{7}$

(G) 1

(H) $\frac{7}{3}$

(I) 3



7. Which statement is the inverse of the following statement?

If $\overline{PQ}$ is a midsegment of $\triangle ABC$, then $\overline{PQ}$ is parallel to a side of $\triangle ABC$.

(A) If $\overline{PQ}$ is a midsegment of $\triangle ABC$, then $\overline{PQ}$ is not parallel to a side of $\triangle ABC$.

(B) If $\overline{PQ}$ is not a midsegment of $\triangle ABC$, then $\overline{PQ}$ is not parallel to a side of $\triangle ABC$.

(C) If $\overline{PQ}$ is not parallel to a side of $\triangle ABC$, then $\overline{PQ}$ is not a midsegment of $\triangle ABC$.

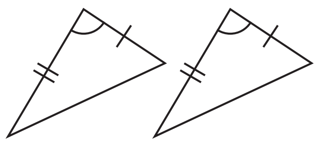
(D) If $\overline{PQ}$ is parallel to a side of $\triangle ABC$, then $\overline{PQ}$ is a midsegment of $\triangle ABC$.

8. $\overline{AB}$ has endpoints $A(0, -4)$ and $B(8, -2)$. What is the slope-intercept form of the equation of the perpendicular bisector of $\overline{AB}$?

(F) $y = -4x + 13$ (H) $y + 3 = -4(x - 4)$

(G) $y = 4x - 19$ (I) $y + 3 = 4(x - 4)$

9. How can you prove that the two triangles at the right are congruent?



(A) ASA (C) SAS

(B) SSS (D) HL

10. Which statement contradicts the statement $\triangle ABC \cong \triangle JMK$ by SAS?

(F) $\angle A$ and $\angle J$ are vertical angles.

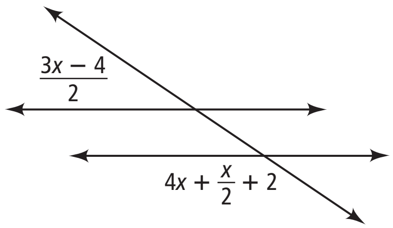
(G) $\overline{BC}$ is the hypotenuse of $\triangle ABC$.

(H) $\triangle ABC$ is isosceles and $\triangle JMK$ is scalene.

(I) $\overline{AB}$ is not congruent to $\overline{MK}$.

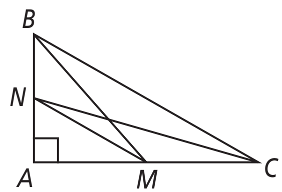
Constructed Response

11. What is the value of x?



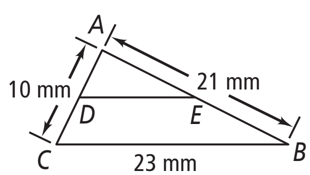
12. $A(-2, -2)$ and $B(5, 7)$ are the endpoints of $\overline{AB}$. Point C lies on $\overline{AB}$ and is $\frac{1}{3}$ of the way from B to A. What is the y-coordinate of Point C?

13. $\triangle ABC$ is a right triangle with area 14 in.^{2}. $\overline{BM}$ and $\overline{CN}$ are medians and $BN = 2$ in. What is the area of $\triangle CNM$ in square inches?



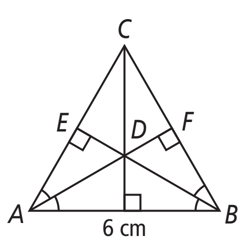
14. The measure of one angle of an isosceles triangle is 70. What is the measure of the largest angle?

15. $\overline{DE}$ is a midsegment of $\triangle ABC$. In millimeters, what is DE?



16. What is the area in square units of a rectangle with vertices $(-2, 5)$, $(3, 5)$, $(3, -1)$, and $(-2, -1)$?

17. In the figure below, the area of $\triangle ADB$ is 4.5 cm^2. In centimeters, what is DF?

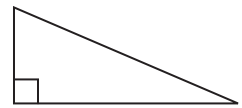


18. If two lines are parallel, then they do not intersect. If two lines do not intersect, then they do not have any points in common.

a. Suppose $\ell \parallel m$. What conclusion can you make about lines ℓ and m?

b. Explain how you arrived at your conclusion.

19. Copy the triangle below.



a. Construct a congruent triangle.

b. Describe your method.

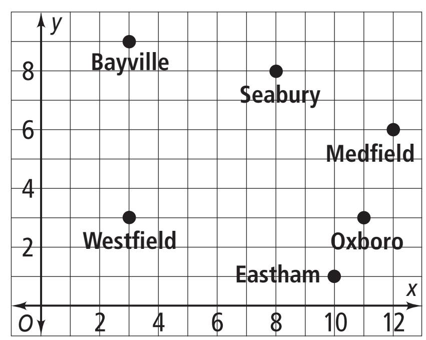
Extended Response

20. Write an indirect proof.

Given: $\triangle ABC$ is obtuse.

Prove: $\triangle ABC$ is not a right triangle.

21. The towns of Westfield, Bayville, and Oxboro need a cell phone tower. The strength of the signal from the tower should be the same for each of the three towns. The map below shows the location of each town, with each grid square representing 1 square mile.



a. At what coordinates should the new cell phone tower be located? Explain.

b. What is the distance from the new tower to Westfield? To Bayville? To Oxboro?

c. Can any of the other towns shown on the map benefit from the new cell phone tower? Explain.

CHAPTER 6

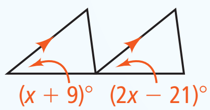
Get Ready!

Lesson 3-2

Properties of Parallel Lines

Algebra **Use properties of parallel lines to find the value of *x*.**

1.



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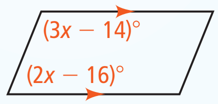

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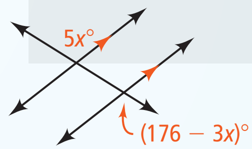
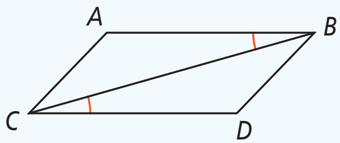


Lesson 3-3

Proving Lines Parallel

Algebra **Determine whether $\overline{AB}$ is parallel to $\overline{CD}$.**

4.



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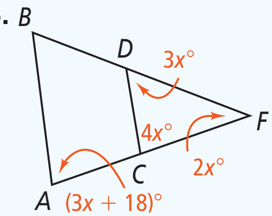

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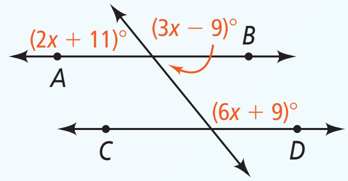


Lesson 3-8

Using Slope to Determine Parallel and Perpendicular Lines

Algebra **Determine whether each pair of lines is *parallel*, *perpendicular*, or *neither*.**

7. $y = -2x;\ y = -2x + 4$

8. $y = -\frac{3}{5}x + 1;\ y = \frac{5}{3}x - 3$

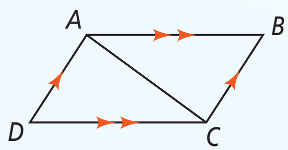
9. $2x - 3y = 1;\ 3x - 2y = 8$

Lessons 4-2 and 4-3

Proving Triangles Congruent

Determine the postulate or theorem that makes each pair of triangles congruent.

10.



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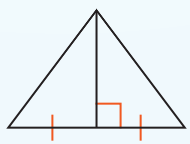

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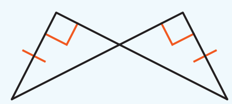

Looking Ahead Vocabulary

13. You know the meaning of *equilateral*. What do you think an *equiangular* polygon is?

14. Think about what a *kite* looks like. What characteristics might a *kite* in geometry have?

15. When a team wins two *consecutive* gold medals, it means they have won two gold medals in a row. What do you think two *consecutive* angles in a quadrilateral means?